Communication Complexity of Combinatorial Auctions with Additional Succinct Bidders

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Combinatorial Auctions

Items $M = [m]$, bidders with valuations $v_1, \dots, v_n : 2^M \to \mathbb{R}_+$

Want to find allocation $(A_1, \dots, A_n)$ optimizing welfare $\sum_i v_i(A_i)$

Algorithm is an **α-approximation** if in expectation (over algorithm's randomness), obtains welfare within an α factor of optimal

Resource constraint: algorithm runs in polynomial **communication**

Why Study CA?

If bidders are cooperative, we know good approximation algorithms

If bidders are **strategic** (aim to maximize their own utility), we need to compute **payments** to incentivize following the algorithm

An algorithm that computes an allocation and incentivizing payments is called a **truthful mechanism***

*We will only be talking about **deterministic** truthful mechanisms

Why Study CA?

Huge gap in what we know between the two settings; for example, when bidders have subadditive (SA) valuations,

- 2-approximation algorithm (and this is optimal)
- Best known truthful mechanism is only a $\sqrt{m/\log m}$-approximation

No separation results (i.e., we have not even ruled out the possibility of a truthful 2-approximation for SA)

Succinct Bidders

A valuation is **succinct** if it can be stated in poly communication

- Example: single-minded (SM) valuation is parameterized by S, x and gets value x if they get a superset of S

If **every** bidder is succinct, we can trivially maximize the welfare

Additional Succinct Bidders

What if we have SA bidders **and** succinct bidders? (e.g., SA ∪ SM)

[RTWZ'24] shows that when you have 3 bidders from SA ∪ SM, **truthful mechanisms** cannot beat a $(1+\sqrt{3})$-approximation

Easy 2-approximation **algorithm**: ask who is SM, guess if SM bidders get their sets, and run 2-approx algorithm on SA bidders

Can we get a 2-approximation for all n?

Results

A 3-approximation hardness for SA ∪ SM, and matching algorithm

A 2-approximation hardness for XOS ∪ SM, and matching algorithm

Separations between SA and SA ∪ SM (XOS and XOS ∪ SM) for all numbers of SA (XOS) bidders

New notion of strong inapproximability with results for SA

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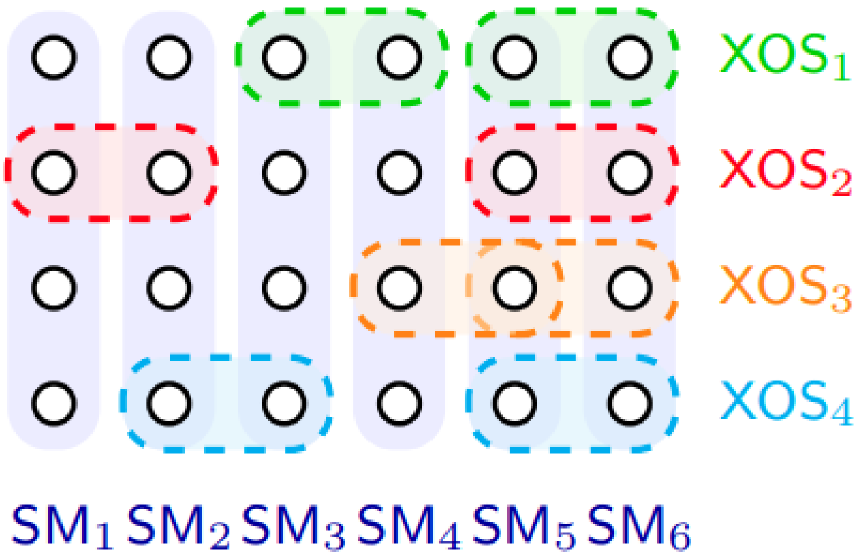
XOS U SM Hard Instance

n XOS bidders and c SM bidders

Each SM bidder likes a column

Each XOS bidder likes many random small sets

There may be a single planted column-set that all the XOS bidders like

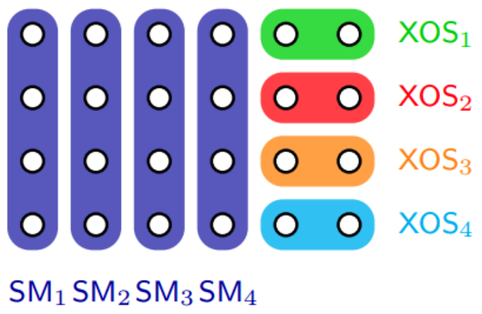


XOS U SM Hard Instance

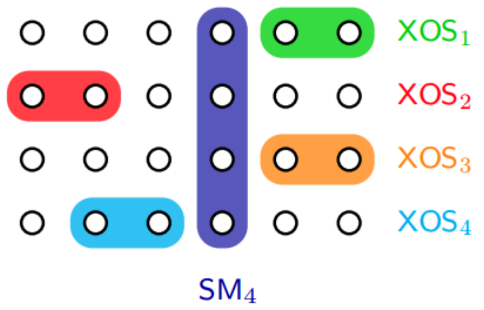
Hard to determine if planted set exists

If it does, optimal welfare is close to 2

If not, optimal welfare is close to 1



(b) Optimal Allocation



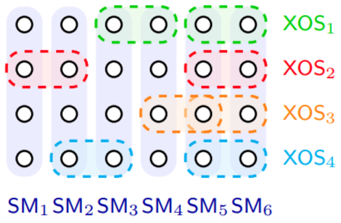
(c) Suboptimal Allocation

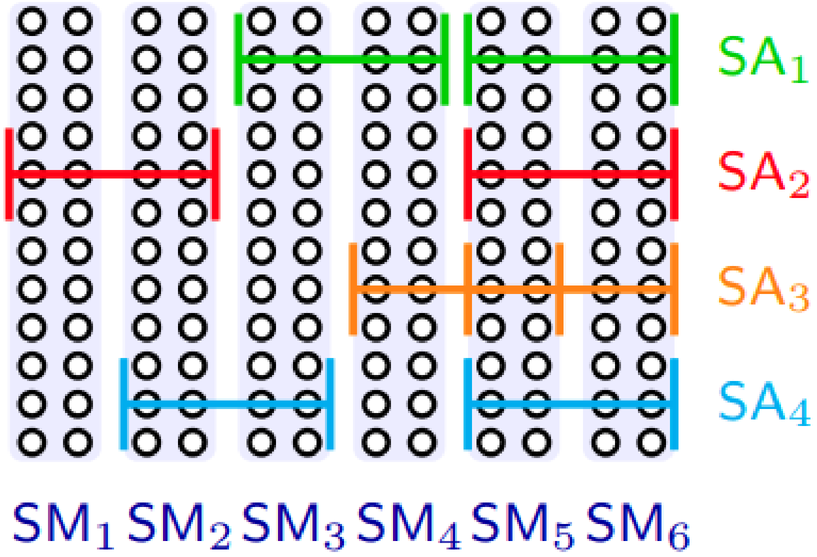
SA ∪ SM Hard Instance

Many more items in each column

Embed hard (to 2-approximate) SA instance in each column

SA bidders constitute welfare 2, SM bidders constitute welfare 1

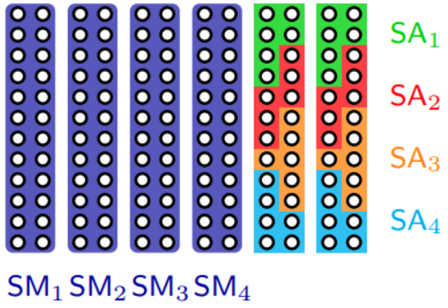




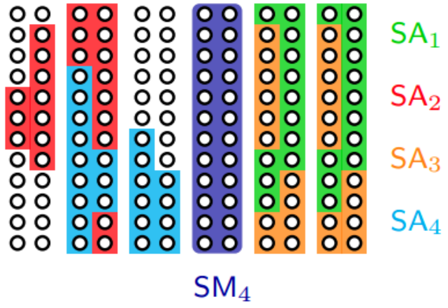
SA U SM Hard Instance

Due to column instances, can recover welfare 1 from SA bidders at best

Since planted set hard to find, SA and SM bidders cannot both be satisfied



(b) Optimal Allocation



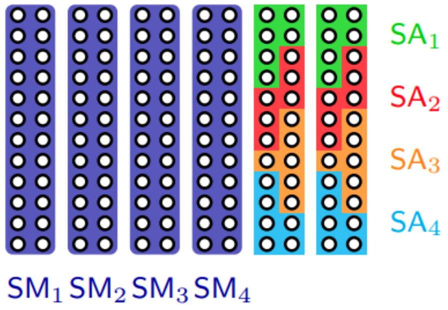
(c) Suboptimal Allocation

SA U SM Hard Instance

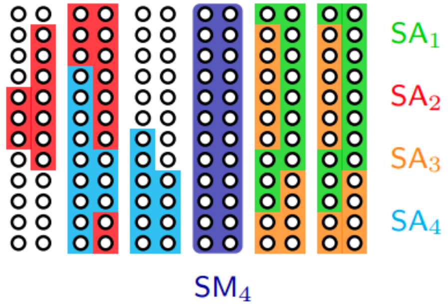
Need to be careful: since each SA bidder only likes small column-sets, you can "spread" them out across columns

Need resulting sparse instances to still be hard to 2-approximate

New "strong inapproximability" notion



(b) Optimal Allocation



(c) Suboptimal Allocation

Conclusion

Succinct bidders can make a **communication** problem harder

Succinct bidders still have promise for separation results, but more innovation is needed to push lower bounds for deterministic truthful mechanisms further

Thank You!

QUESTIONS?